MAT 5-236 Exam 1 Solutions.

1.a Model the amount of salt. See text.

b. $\frac{da}{dt} = -\gamma ab$

$\frac{db}{dt} = -\gamma ab$

c. $\frac{dP}{dt} = r \cdot P - A \qquad P(t_0) = B$

d. $\frac{dx}{dt} = ax + bxy$

$\frac{dy}{dt} = -cy + dxy$

other single species growth terms also OK

Change 5's to 7's

2. $\frac{dy}{dt} = y(5-y) \quad 0 < y < 5 \qquad \int \frac{dy}{y(5-y)} = \int dt$

Partial fractions: $\frac{1}{5}\int \frac{1}{y} + \frac{1}{5-y}\, dy = \frac{1}{5}(\ln y - \ln 5-y) = \frac{1}{5}\ln \frac{y}{5-y}$

LHS

RHS $= t + C$ so $\ln \frac{y}{5-y} = 5t + C_1$ so $\frac{y}{5-y} = C_2 e^{5t}$

Solve for $y = \frac{5C_2 e^{5t}}{1 + C_1 e^{5t}}$ $y(0) = 1$ so $C_2 = \frac{1}{4}$ $y(t) = \frac{5/4\, e^{5t}}{1 + \frac{1}{4}e^{5t}}$

3. Euler RHS: $3t - y \qquad \Delta t = .1$

1. $y_0 = y(1) = 2 \qquad y_n = y_{n-1} + f(t_{n-1}, y_{n-1})\,\Delta t$

$y_1 = y(1.1) = 2 + (3-2)\cdot .1 = 2.1$

$y_2 = y(1.2) = 2.1 + (3\cdot 1.1 - 2.1)\cdot .1 = 2.1 + .12 = 2.22$

4. 1. Put in form $\frac{dy}{dt} + g(t)y = f(t)$ $\quad \frac{dy}{dt} - \frac{2t}{1+t^2}y = 3$

2. Find integrating factor $\mu(t) = e^{\int g(t)\,dt}$:

$\int \frac{-2t}{1+t^2}dt = -\int \frac{1}{u}du = -\ln|u| = -\ln(1+t^2)$

$u = 1+t^2$

$du = 2t$

$\mu(t) = e^{-\ln(1+t^2)}$

$= \frac{1}{1+t^2}$

3. Mult both sides by I.F. Adjust LHS using prod. rule

$\frac{d(\mu(t)y)}{dt} = \mu(t)f(t)$ & integrate

$y = \frac{\int \frac{3}{1+t^2}}{\frac{1}{1+t^2}} = (3\arctan t + c)(1+t^2)$

5 $\frac{dy}{dt} = 3y + 2e^{3t}$ $\quad y(0) = -1$

a) $y_h = e^{3t}$ b) y_p guess qte^{3t}

$qe^{3t} + 3qte^{3t} = 3qte^{3t} + 2e^{3t}$

$q = 2$

$y_p = 2te^{3t}$

c) General solution: $y(t) = c \cdot e^{3t} + 2te^{3t}$

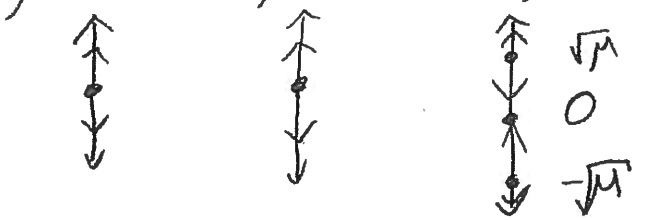
$-1 = y(0) = c$

6.

7. $dy/dt = y^3 - \mu y = y(y^2 - \mu)$

μ is the only bifurcation pt.

$\mu < 0$ $\quad \mu = 0$ $\quad \mu > 0$



8. a. $x'(t) = v(t)$

$v'(t) = -4x$

damped harmonic oscillation

8b to follow

9. a. See p 175 (9 0), (0, 3), (2, 0), (1, 1) competitive species

10. To follow.

a. false $\sin \pi x$ $+ \to -$ at 1 so sink. b. True. Unig. theorem.

$\frac{\partial f}{\partial y}$ cont ✓

c. True d. false e. true. linear

a) 7 b) 8 c) 5 d) 6

12.