1. **(a)** The characteristic polynomial is

$$(3-\lambda)(-2-\lambda)=0,$$

and therefore the eigenvalues are $\lambda_1=-2$ and $\lambda_2=3$.

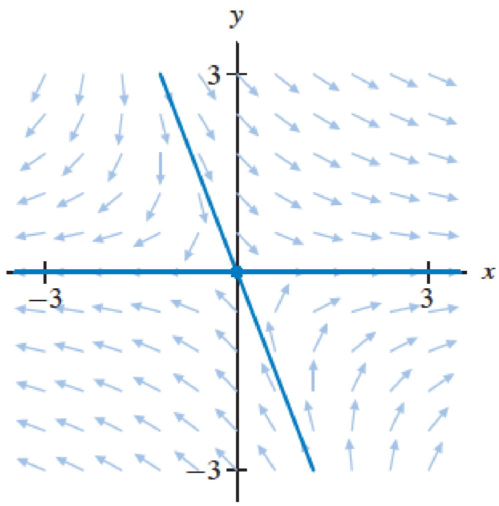
(b) To obtain the eigenvectors (x_1, y_1) for the eigenvalue $\lambda_1=-2$, we solve the system of equations

$$\begin{cases} 3x_1+2y_1=-2x_1 \\ -2y_1=-2y_1 \end{cases}$$

and obtain $5x_1=-2y_1$.

Using the same procedure, we see that the eigenvectors (x_2, y_2) for $\lambda_2=3$ must satisfy the equation $y_2=0$.

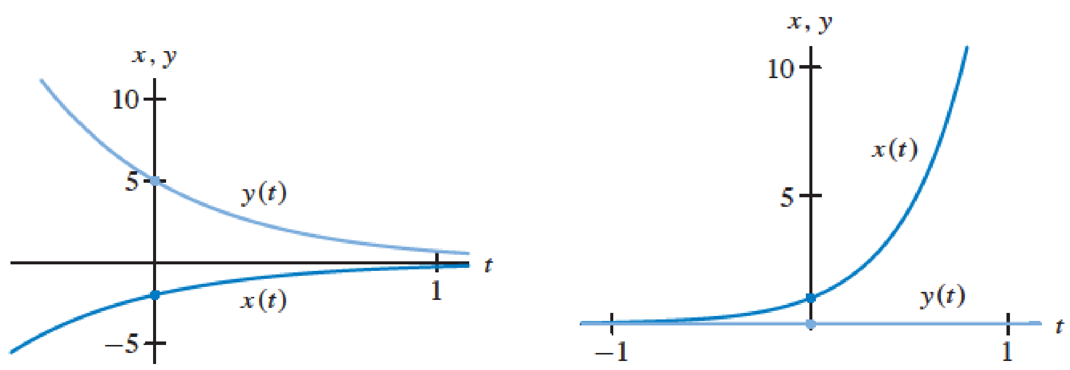
(c)



(d) One eigenvector $\mathbf{V}_1$ for λ_1 is $\mathbf{V}_1 = (-2, 5)$, and one eigenvector $\mathbf{V}_2$ for λ_2 is $\mathbf{V}_2 = (1, 0)$.

Given the eigenvalues and these eigenvectors, we have the two linearly independent solutions

$$\mathbf{Y}_1(t) = e^{-2t} \begin{pmatrix} -2 \\ 5 \end{pmatrix} \quad \text{and} \quad \mathbf{Y}_2(t) = e^{3t} \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$



The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$. The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.

(e) The general solution to this linear system is

$$\mathbf{Y}(t) = k_1 e^{-2t} \begin{pmatrix} -2 \\ 5 \end{pmatrix} + k_2 e^{3t} \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

2. **(a)** The characteristic polynomial is

$$(-4-\lambda)(-3-\lambda)-2=\lambda^2+7\lambda+10=0,$$

and therefore the eigenvalues are $\lambda_1=-2$ and $\lambda_2=-5$.

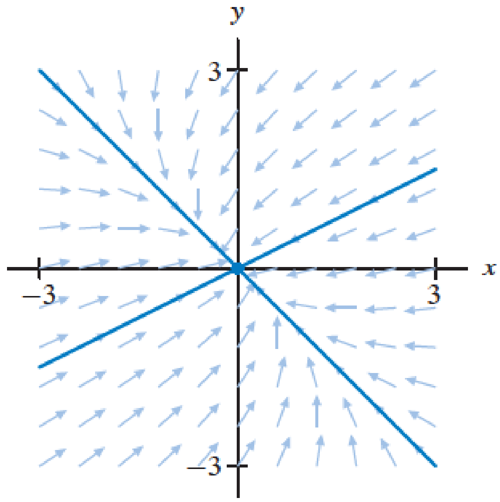
(b) To obtain the eigenvectors (x_1, y_1) for the eigenvalue $\lambda_1=-2$, we solve the system of equations

$$\begin{cases} -4x_1-2y_1=-2x_1 \\ -x_1-3y_1=-2y_1 \end{cases}$$

and obtain $y_1=-x_1$.

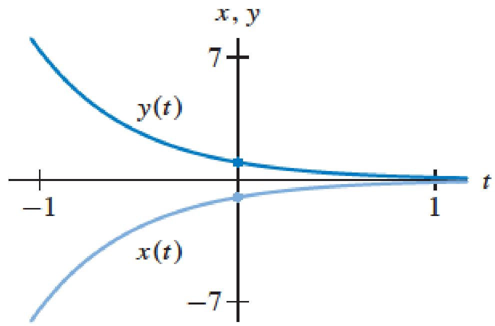
Using the same procedure, we obtain the eigenvectors (x_2, y_2) where $x_2=2y_2$ for $\lambda_2=-5$.

(c)

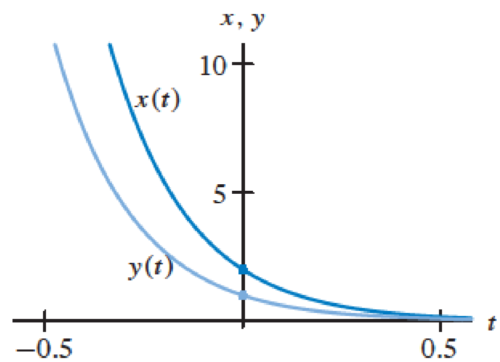


(d) One eigenvector $\mathbf{V}_1$ for λ_1 is $\mathbf{V}_1 = (1, -1)$, and one eigenvector $\mathbf{V}_2$ for λ_2 is $\mathbf{V}_2 = (2, 1)$.
Given the eigenvalues and these eigenvectors, we have two linearly independent solutions

$$\mathbf{Y}_1(t) = e^{-2t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \text{and} \quad \mathbf{Y}_2(t) = e^{-5t}\begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$



The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$.



The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.

(e) The general solution to this linear system is

$$\mathbf{Y}(t) = k_1 e^{-2t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + k_2 e^{-5t}\begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

3. **(a)** The eigenvalues are the roots of the characteristic polynomial, so they are the solutions of

$$(-5-\lambda)(-4-\lambda)-2=\lambda^2+9\lambda+18=0.$$

Therefore, the eigenvalues are $\lambda_1=-3$ and $\lambda_2=-6$.

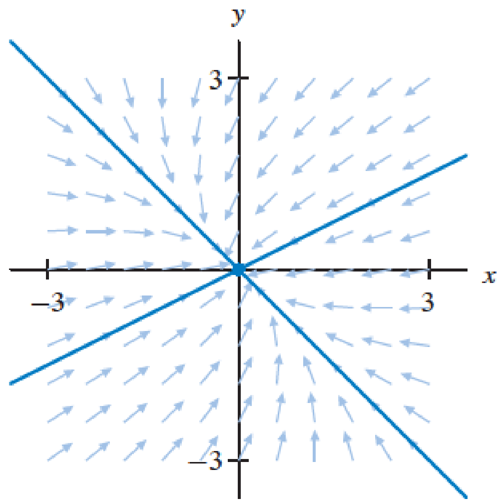
(b) To obtain the eigenvectors (x_1, y_1) for the eigenvalue $\lambda_1=-3$, we solve the system of equations

$$\begin{cases} -5x_1-2y_1=-3x_1 \\ -x_1-4y_1=-3y_1 \end{cases}$$

and obtain $y_1=-x_1$.

Using the same procedure, we obtain the eigenvalues (x_2, y_2) where $x_2=2y_2$ for $\lambda_2=-6$.

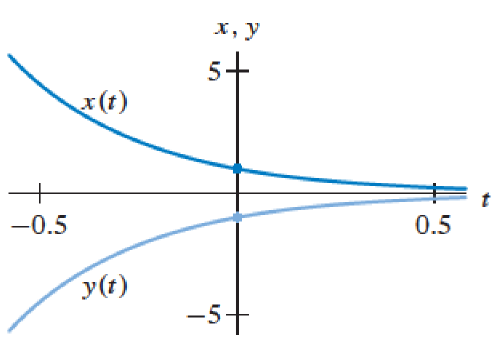
(c)



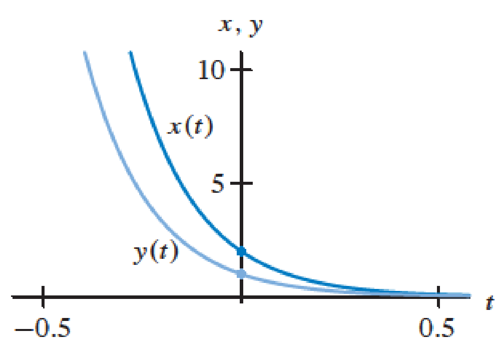
(d) One eigenvector $\mathbf{V}_1$ for $\lambda_1=-3$ is $\mathbf{V}_1=(1,-1)$, and one eigenvector $\mathbf{V}_2$ for $\lambda_2=-6$ is $\mathbf{V}_2=(2,1)$.

Given the eigenvalues and these eigenvectors, we have two linearly independent solutions

$$\mathbf{Y}_1(t)=e^{-3t}\begin{pmatrix}1\\-1\end{pmatrix} \quad \text{and} \quad \mathbf{Y}_2(t)=e^{-6t}\begin{pmatrix}2\\1\end{pmatrix}.$$



The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$.



the $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.

(e) The general solution to this linear system is

$$\mathbf{Y}(t)=k_1e^{-3t}\begin{pmatrix}1\\-1\end{pmatrix}+k_2e^{-6t}\begin{pmatrix}2\\1\end{pmatrix}.$$

4. **(a)** The characteristic polynomial is

$$(2-\lambda)(4-\lambda)+1=\lambda^2-6\lambda+9=0,$$

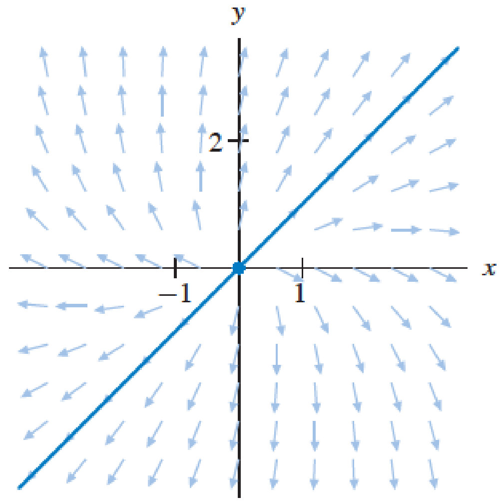
and therefore there is only one eigenvalue, $\lambda=3$.

(b) To obtain the eigenvectors (x_1, y_1) for the eigenvalue $\lambda=3$, we solve the system of equations

$$\begin{cases} 2x_1+y_1=3x_1 \\ -x_1+4y_1=3y_1 \end{cases}$$

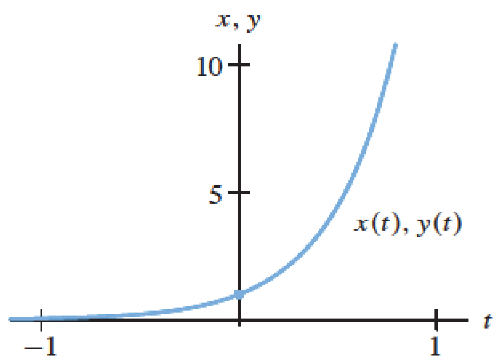
and obtain $y_1=x_1$.

(c)



(d) One eigenvector **V** for λ is $\mathbf{V}=(1,1)$. Given this eigenvector, we have the solution

$$\mathbf{Y}(t)=e^{3t}\begin{pmatrix}1\\1\end{pmatrix}.$$



The $x(t)$- and $y(t)$-graphs (which are identical) for $\mathbf{Y}(t)$

(e) Since the method of eigenvalues and eigenvectors does not give us a second solution that is linearly independent from $\mathbf{Y}(t)$, we cannot form the general solution.

5. **(a)** The characteristic polynomial is

$$\left(-\tfrac{1}{2} - \lambda\right)^2 = 0,$$

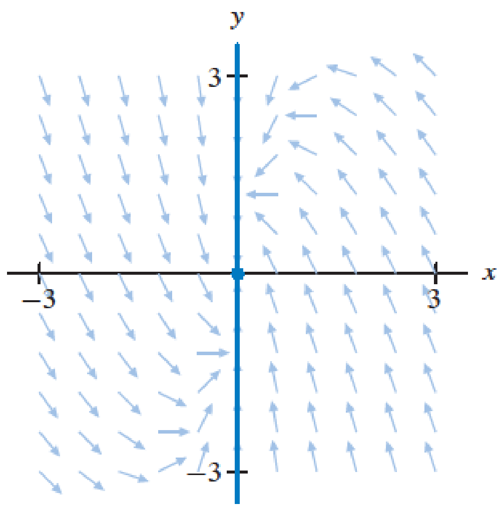
and therefore there is only one eigenvalue, $\lambda = -1/2$.

(b) To obtain the eigenvectors (x_1, y_1) for the eigenvalue $\lambda = -1/2$, we solve the system of equations

$$\begin{cases} -\tfrac{1}{2}x_1 = -\tfrac{1}{2}x_1 \\ x_1 - \tfrac{1}{2}y_1 = -\tfrac{1}{2}y_1 \end{cases}$$

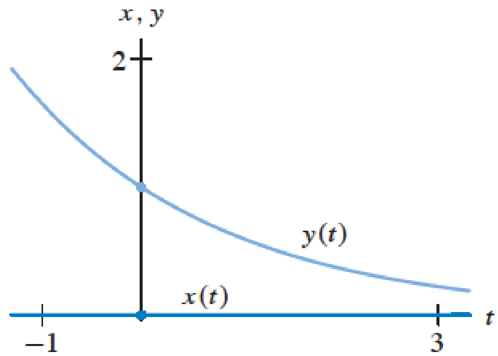
and obtain $x_1 = 0$.

(c)



(d) Given the eigenvalue $\lambda = -1/2$ and the eigenvector $\mathbf{V} = (0, 1)$, we have the solution

$$\mathbf{Y}(t) = e^{-t/2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$



The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}(t)$.

(e) Since the method of eigenvalues and eigenvectors does not give us a second solution that is linearly independent from $\mathbf{Y}(t)$, we cannot form the general solution.

6. **(a)** The characteristic polynomial is

$$(5-\lambda)(-\lambda) - 36 = 0,$$

and therefore the eigenvalues are $\lambda_1 = -4$ and $\lambda_2 = 9$.

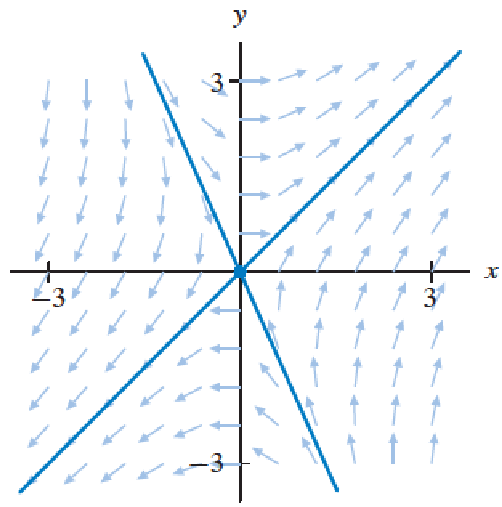
(b) To obtain the eigenvectors (x_1, y_1) for the eigenvalue $\lambda_1 = -4$, we solve the system of equations

$$\begin{cases} 5x_1 + 4y_1 = -4x_1 \\ 9x_1 = -4y_1 \end{cases}$$

and obtain $9x_1 = -4y_1$.

Using the same procedure, we see that the eigenvectors (x_2, y_2) for $\lambda_2 = 9$ must satisfy the equation $y_2 = x_2$.

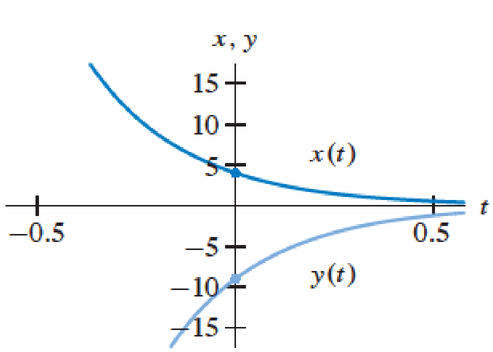
(c)

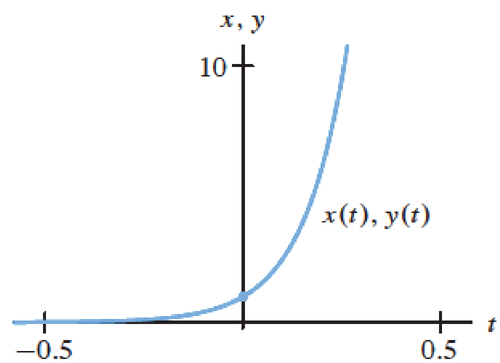


(d) One eigenvector $\mathbf{V}_1$ for λ_1 is $\mathbf{V}_1 = (4, -9)$, and one eigenvector $\mathbf{V}_2$ for λ_2 is $\mathbf{V}_2 = (1, 1)$.

Given the eigenvalues and these eigenvectors, we have the two linearly independent solutions

$$\mathbf{Y}_1(t) = e^{-4t}\begin{pmatrix} 4 \\ -9 \end{pmatrix} \quad \text{and} \quad \mathbf{Y}_2(t) = e^{9t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$





The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$.

The (identical) $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.

(e) The general solution to this linear system is

$$\mathbf{Y}(t) = k_1 e^{-4t}\begin{pmatrix} 4 \\ -9 \end{pmatrix} + k_2 e^{9t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

7. **(a)** The characteristic polynomial is

$$(3-\lambda)(-\lambda)-4=\lambda^2-3\lambda-4=(\lambda-4)(\lambda+1)=0,$$

and therefore the eigenvalues are $\lambda_1=-1$ and $\lambda_2=4$.

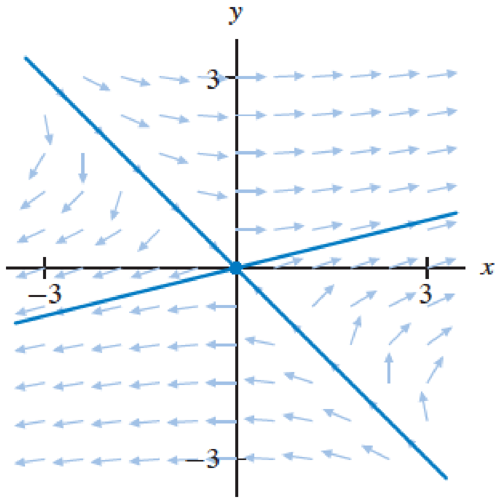
(b) To obtain the eigenvectors (x_1, y_1) for the eigenvalue $\lambda_1=-1$, we solve the system of equations

$$\begin{cases} 3x_1+4y_1=-x_1 \\ x_1=-y_1 \end{cases}$$

and obtain $y_1=-x_1$.

Using the same procedure, we obtain the eigenvectors (x_2, y_2) where $x_2=4y_2$ for $\lambda_2=4$.

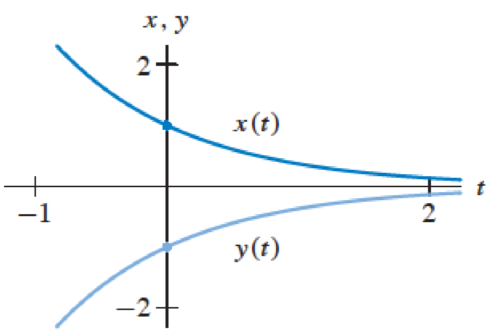
(c)



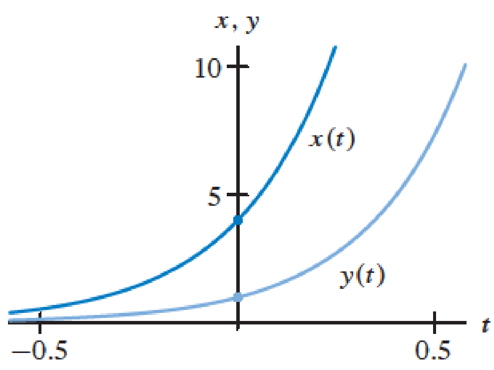
(d) One eigenvector $\mathbf{V}_1$ for λ_1 is $\mathbf{V}_1=(1,-1)$, and one eigenvector $\mathbf{V}_2$ for λ_2 is $\mathbf{V}_2=(4,1)$.

Given the eigenvalues and these eigenvectors, we have two linearly independent solutions

$$\mathbf{Y}_1(t)=e^{-t}\begin{pmatrix}1\\-1\end{pmatrix} \quad \text{and} \quad \mathbf{Y}_2(t)=e^{4t}\begin{pmatrix}4\\1\end{pmatrix}.$$



The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$.



The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.

(e) The general solution to this linear system is

$$\mathbf{Y}(t)=k_1e^{-t}\begin{pmatrix}1\\-1\end{pmatrix}+k_2e^{4t}\begin{pmatrix}4\\1\end{pmatrix}$$

8. **(a)** The characteristic polynomial is

$$(2-\lambda)(1-\lambda)-1=\lambda^2-3\lambda+1=0,$$

and therefore the eigenvalues are

$$\lambda_1=\frac{3+\sqrt{5}}{2} \quad \text{and} \quad \lambda_2=\frac{3-\sqrt{5}}{2}.$$

(b) To obtain the eigenvectors (x_1, y_1) for the eigenvalue $\lambda_1=(3+\sqrt{5})/2$, we solve the system of equations

$$\begin{cases} 2x_1-y_1=\dfrac{3+\sqrt{5}}{2}x_1 \\ -x_1+y_1=\dfrac{3+\sqrt{5}}{2}y_1 \end{cases}$$

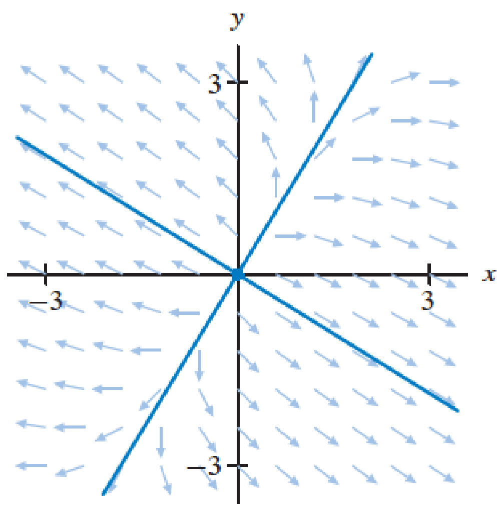
and obtain

$$y_1=\frac{1-\sqrt{5}}{2}x_1,$$

which is equivalent to the equation $2y_1=(1-\sqrt{5})x_1$.

Using the same procedure, we obtain the eigenvectors (x_2, y_2) where $2y_2=(1+\sqrt{5})x_2$ for $\lambda_2=(3-\sqrt{5})/2$.

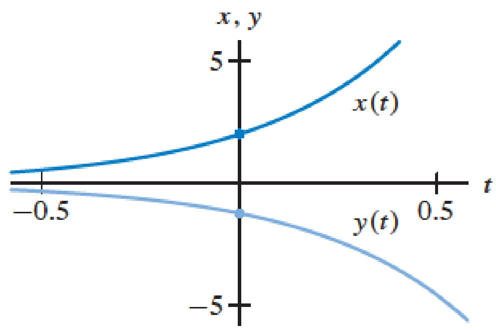
(c)



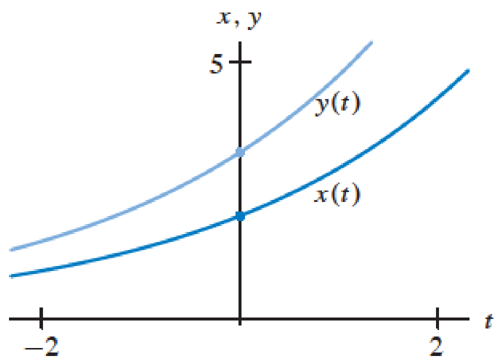
(d) One eigenvector $\mathbf{V}_1$ for the eigenvalue λ_1 is $\mathbf{V}_1 = (2, 1-\sqrt{5})$, and one eigenvector $\mathbf{V}_2$ for the eigenvalue λ_2 is $\mathbf{V}_2 = (2, 1+\sqrt{5})$.

Given the eigenvalues and these eigenvectors, we have two linearly independent solutions

$$\mathbf{Y}_1(t) = e^{(3+\sqrt{5})t/2}\begin{pmatrix} 2 \\ 1-\sqrt{5} \end{pmatrix} \quad \text{and} \quad \mathbf{Y}_2(t) = e^{(3-\sqrt{5})t/2}\begin{pmatrix} 2 \\ 1+\sqrt{5} \end{pmatrix}.$$



The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$.



The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.

(e) The general solution to this linear system is

$$\mathbf{Y}(t) = k_1 e^{(3+\sqrt{5})t/2}\begin{pmatrix} 2 \\ 1-\sqrt{5} \end{pmatrix} + k_2 e^{(3-\sqrt{5})t/2}\begin{pmatrix} 2 \\ 1+\sqrt{5} \end{pmatrix}.$$

9. **(a)** The characteristic polynomial is

$$(2-\lambda)(1-\lambda)-1=\lambda^2-3\lambda+1=0,$$

and therefore the eigenvalues are

$$\lambda_1=\frac{3+\sqrt{5}}{2} \quad \text{and} \quad \lambda_2=\frac{3-\sqrt{5}}{2}.$$

(b) To obtain the eigenvectors (x_1, y_1) for the eigenvalue $\lambda_1=(3+\sqrt{5})/2$, we solve the system of equations

$$\begin{cases} 2x_1+y_1=\dfrac{3+\sqrt{5}}{2}x_1 \\ x_1+y_1=\dfrac{3+\sqrt{5}}{2}y_1 \end{cases}$$

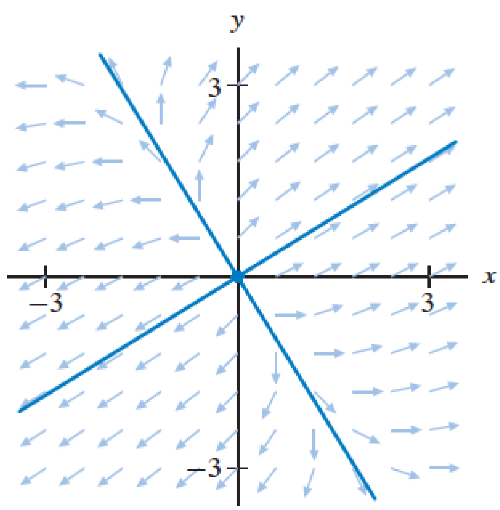
and obtain

$$y_1=\frac{-1+\sqrt{5}}{2}x_1,$$

which is equivalent to the equation $2y_1=(-1+\sqrt{5})x_1$.

Using the same procedure, we obtain the eigenvectors (x_2, y_2) where $2y_2=(-1-\sqrt{5})x_2$ for $\lambda_2=(3-\sqrt{5})/2$.

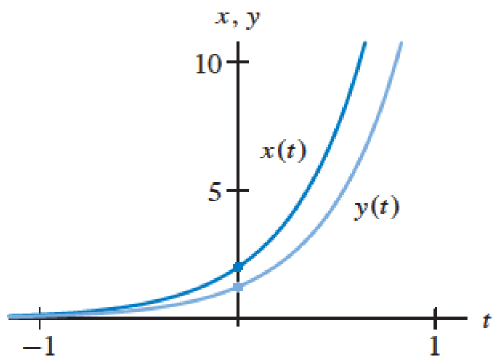
(c)



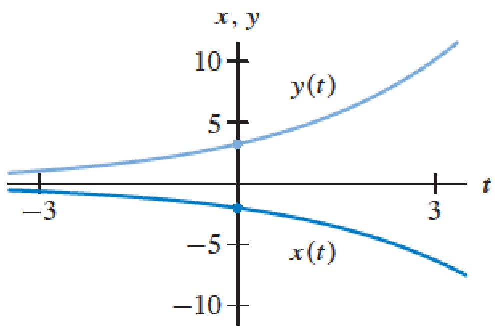
(d) One eigenvector $\mathbf{V}_1$ for the eigenvalue λ_1 is $\mathbf{V}_1 = (2, -1+\sqrt{5})$, and one eigenvector $\mathbf{V}_2$ for the eigenvalue λ_2 is $\mathbf{V}_2 = (-2, 1+\sqrt{5})$.

Given the eigenvalues and these eigenvectors, we have two linearly independent solutions

$$\mathbf{Y}_1(t) = e^{(3+\sqrt{5})t/2} \begin{pmatrix} 2 \\ -1+\sqrt{5} \end{pmatrix} \quad \text{and} \quad \mathbf{Y}_2(t) = e^{(3-\sqrt{5})t/2} \begin{pmatrix} -2 \\ 1+\sqrt{5} \end{pmatrix}.$$



The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$.



The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.

(e) The general solution to this linear system is

$$\mathbf{Y}(t) = k_1 e^{(3+\sqrt{5})t/2} \begin{pmatrix} 2 \\ -1+\sqrt{5} \end{pmatrix} + k_2 e^{(3-\sqrt{5})t/2} \begin{pmatrix} -2 \\ 1+\sqrt{5} \end{pmatrix}.$$

10. **(a)** The characteristic polynomial is

$$(-1-\lambda)(-4-\lambda)+2=\lambda^2+5\lambda+6=(\lambda+3)(\lambda+2)=0,$$

and therefore the eigenvalues are $\lambda_1=-2$ and $\lambda_2=-3$.

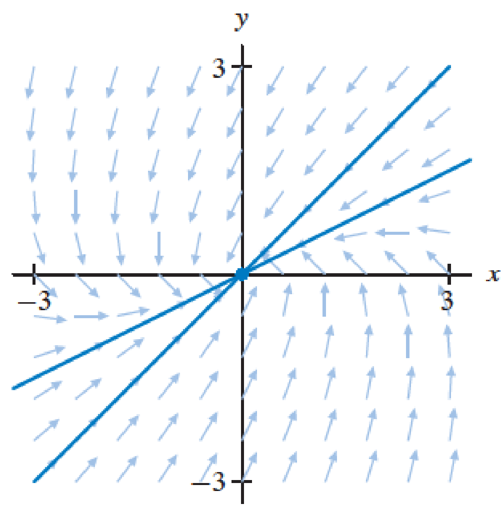
(b) To obtain the eigenvectors (x_1, y_1) for the eigenvalue $\lambda_1=-2$, we solve the system of equations

$$\begin{cases} -x_1-2y_1=-2x_1 \\ x_1-4y_1=-2y_1 \end{cases}$$

and obtain $x_1=2y_1$.

Using the same procedure, we obtain the eigenvectors (x_2, y_2) where $x_2=y_2$ for $\lambda_2=-3$.

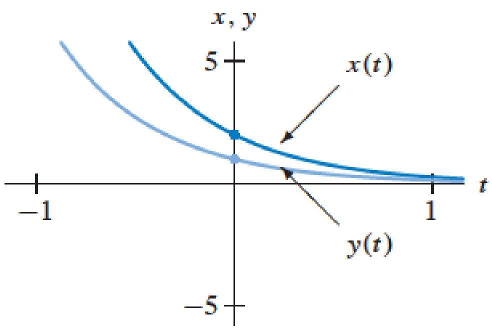
(c)



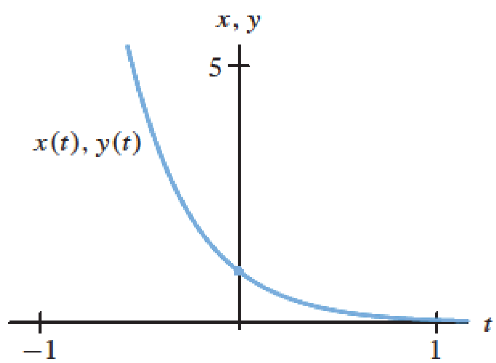
(d) One eigenvector $\mathbf{V}_1$ for λ_1 is $\mathbf{V}_1 = (2, 1)$, and one eigenvector $\mathbf{V}_2$ for λ_2 is $\mathbf{V}_2 = (1, 1)$.

Given the eigenvalues and these eigenvectors, we have two linearly independent solutions

$$\mathbf{Y}_1(t) = e^{-2t}\begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \text{and} \quad \mathbf{Y}_2(t) = e^{-3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$



The $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_1(t)$.



The identical) $x(t)$- and $y(t)$-graphs for $\mathbf{Y}_2(t)$.

(e) The general solution to this linear system is

$$\mathbf{Y}(t) = k_1 e^{-2t}\begin{pmatrix} 2 \\ 1 \end{pmatrix} + k_2 e^{-3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$