MAT8-236 Differential Equations

Exam 2 April 22, 2008

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name

Closed book and notes. Graphing calculators are allowed, but not laptop computers or calculators capable of symbolic mathematics. Do not store and use formulas, algorithms, or other information in your calculator. Write your answers on the paper provided then staple these pages to front of your exam. On each problem justify your answers and show your work.

1. (10 pts.) Solve the initial value problem: $y'' + 4y' + 20y = 0$, $y(0) = 2$, $y'(0) = -8$

2. (12 pts.) The behavior of a damped harmonic oscillator is determined by the mass of the system (m), the spring constant (k), and the damping constant (b). The behavior of such a physical system can be characterized as under-damped, critically damped, or over-damped. Give the differential equation and explain the difference between these three behaviors and give values of m, k, and b that will result in each. Justify your examples.

3. (10 pts.) Solve the differential equation $y''+ 3y' + 2y = \cos(t)$. What is the long-term behavior of this system?

4. (6 pts.) Consider the system $dY/dt = AY$. Mathematica reveals that the eigenvalues and associated eigenvectors of A are 2 with (1,1) and - 4 with (0,1) . Use this information to produce the general solution to this system. Also, give its phase portrait.

5. (12 pts.) Let x(t) represent profit from Paul's CD store and y(t) represent profit from Bob's nearby CD store. Consider the following system:

$x' = 2x + 2y$

$y' = 1x + 3y$

a. What is the interplay of profits based on this system? Explain.

b. Find the eigenvalues and eigenvectors of the coefficient matrix.

c. Give the general form of the solution.

d. What does this model predict about sales for Paul? For Bob?

6. (15 pts.) Consider the following system:
$x' = -3x + 1y$
$y' = 3x - 1y$

a. Find the eigenvalues and eigenvectors of the coefficient matrix.
b. Give the general form of the solution
c. Sketch the curves x(t) and y(t) associated with the initial condition Y(0) = (1,0)
d. Sketch the phase portrait for the system.

7. (14 pts.) Let $\frac{dY}{dt} = \begin{pmatrix} 0 & 2a \\ 1 & a \end{pmatrix} Y$ be a linear system of differential equations. Draw the curve in the trace determinant plane that is obtained from varying the parameter a, determine the bifurcation values of a, and draw and briefly discuss the different types of phase portraits that are exhibited by this one-parameter family.

8. (9 pts). True or false? If it is true, explain why. If it is false, provide a counterexample.

a. The function Y(t) = (sin 3t, cos t) is not the solution for any linear system.

b. For all 2 by 2 linear systems $\frac{dY}{dt} = AY$, the solution curves never have a singularity.
c. (0,0) is an equilibrium point for the equation $y'' + 3y' + 2y = 1$.

9. (12 pts.) Here are four systems and four direction fields. For each system, identify the type of the equilibrium points by name, then match them with the correct direction field. Justify your answers.

a. $x' = x + 2y,\ y' = -2x + y$

b. $x' = x + 2y,\ y' = -2x + y$

c. $x' = 2y,\ y' = 2x$

d. $x' = 2y,\ y' = -2x$