Problem 19 matching

- 19. First, we compute the characteristic polynomials and eigenvalues for each matrix.
 - (i) The characteristic polynomial is $\lambda^2 + 1$, and the eigenvalues are $\lambda = \pm i$. Center.
 - (ii) The characteristic polynomial is $\lambda^2 + 2\lambda 2$, and the eigenvalues are $\lambda = 1 \pm \sqrt{3}$. Saddle.
 - (iii) The characteristic polynomial is $\lambda^2 + 3\lambda + 1$, and the eigenvalues are $\lambda = (-3 \pm \sqrt{5})/2$. Sink.
 - (iv) The characteristic polynomial is $\lambda^2 + 1$, and the eigenvalues are $\lambda = \pm i$. Center.
 - (v) The characteristic polynomial is $\lambda^2 \lambda 2$, and the eigenvalues are $\lambda = -1$ and $\lambda = 2$. Saddle.
 - (vi) The characteristic polynomial is $\lambda^2 3\lambda + 1$, and the eigenvalues are $\lambda = (3 \pm \sqrt{5})/2$. Source.
 - (vii) The characteristic polynomial is $\lambda^2 + 4\lambda + 4$. The eigenvalue $\lambda = -2$ is a repeated eigenvalue. Sink.
 - (viii) The characteristic polynomial is $\lambda^2 + 2\lambda + 3$, and the eigenvalues are $\lambda = -1 \pm i\sqrt{2}$. Spiral sink.

Given this information, we can match the matrices with the phase portraits.

- (a) This portrait is a center. There are two possibilities, (i) and (iv). At (1, 0), the vector for (i) is (1, -2), and the vector for (iv) is (-1, -2). This phase portrait corresponds to matrix (iv).
- (b) This portrait is a sink with two lines of eigenvectors. The only possibility is matrix (iii).
- (c) This portrait is a saddle. The only possibilities are (ii) and (v). However, in (v), all vectors on the y-axis are eigenvectors corresponding to the eigenvalue $\lambda = -1$. Therefore, the phase portrait cannot correspond to (v).
- (d) This portrait is a sink with a single line of eigenvectors. The only possibility is matrix (vii).