3. To compute the general solution of the unforced equation, we use the method of Section 3.6. The characteristic polynomial is

$$s^2 - s - 2,$$

so the eigenvalues are $s = -1$ and $s = 2$. Hence, the general solution of the homogeneous equation is

$$k_1 e^{-t} + k_2 e^{2t}.$$

To find a particular solution of the forced equation, we guess $y_p(t) = ke^{3t}$. Substituting into the left-hand side of the differential equation gives

$$\begin{aligned}\frac{d^2 y_p}{dt^2} - \frac{dy_p}{dt} - 2y_p &= 9ke^{3t} - 3ke^{3t} - 2ke^{3t} \\ &= 4ke^{3t}.\end{aligned}$$

In order for $y_p(t)$ to be a solution of the forced equation, we must take $k = 5/4$. The general solution of the forced equation is

$$y(t) = k_1 e^{-t} + k_2 e^{2t} + \tfrac{5}{4} e^{3t}.$$

5. To compute the general solution of the unforced equation, we use the method of Section 3.6. The characteristic polynomial is

$$s^2 + 4s + 13,$$

so the eigenvalues are $s = -2 \pm 3i$. Hence, the general solution of the homogeneous equation is

$$k_1 e^{-2t} \cos 3t + k_2 e^{-2t} \sin 3t.$$

To find a particular solution of the forced equation, we guess $y_p(t) = ke^{-2t}$. Substituting into the left-hand side of the differential equation gives

$$\begin{aligned}\frac{d^2 y_p}{dt^2} + 4\frac{dy_p}{dt} + 13y_p &= 4ke^{-2t} - 8ke^{-2t} + 13ke^{-2t} \\ &= 9ke^{-2t}.\end{aligned}$$

In order for $y_p(t)$ to be a solution of the forced equation, we must take $k = -1/3$. The general solution of the forced equation is

$$y(t) = k_1 e^{-2t} \cos 3t + k_2 e^{-2t} \sin 3t - \tfrac{1}{3} e^{-2t}.$$

17. **(a)** The characteristic polynomial of the unforced equation is

$$s^2 + 4s + 20.$$

So the eigenvalues are $s = -2 \pm 4i$, and the general solution of the unforced equation is

$$k_1 e^{-2t} \cos 4t + k_2 e^{-2t} \sin 4t.$$

To find a particular solution of the forced equation, we guess $y_p(t) = ke^{-2t}$. Substituting $y_p(t)$ into the left-hand side of the differential equation gives

$$\begin{aligned} \frac{d^2 y_p}{dt^2} + 4\frac{dy_p}{dt} + 20y_p &= 4ke^{-2t} - 8ke^{-2t} + 20ke^{-2t} \\ &= 16ke^{-2t}. \end{aligned}$$

So $k = 1/16$ yields a solution of the forced equation.

The general solution of the forced equation is therefore

$$y(t) = k_1 e^{-2t} \cos 4t + k_2 e^{-2t} \sin 4t + \tfrac{1}{16} e^{-2t}.$$

(b) The derivative of the general solution is

$$y'(t) = -2k_1 e^{-2t} \cos 4t - 4k_1 e^{-2t} \sin 4t - 2k_2 e^{-2t} \sin 4t + 4k_2 e^{-2t} \cos 4t - \tfrac{1}{8} e^{-2t}.$$

To find the solution with $y(0) = y'(0) = 0$, we evaluate at $t = 0$ and obtain the simultaneous equations

$$\begin{cases} k_1 + \frac{1}{16} = 0 \\ -2k_1 + 4k_2 - \frac{1}{8} = 0. \end{cases}$$

Solving, we find that $k_1 = -1/16$ and $k_2 = 0$, so the solution of the initial-value problem is

$$y(t) = -\tfrac{1}{16} e^{-2t} \cos 4t + \tfrac{1}{16} e^{-2t}.$$

(c) Every solution tends to zero like e^{-2t} and all but one exponential solution oscillates with frequency $2/\pi$.

19. The natural guesses of $y_p(t) = ke^{-t}$ and $y_p(t) = kte^{-t}$ fail to be solutions of the forced equation because they are both solutions of the unforced equation. (The characteristic polynomial of the unforced equation is

$$s^2 + 2s + 1,$$

which has -1 as a double root.)

So we guess $y_p(t) = kt^2e^{-t}$. Substituting this guess into the left-hand side of the differential equation gives

$$\begin{aligned}\frac{d^2y_p}{dt^2} + 2\frac{dy_p}{dt} + y_p &= (2ke^{-t} - 4kte^{-t} + kt^2e^{-t}) + 2(2kte^{-t} - kt^2e^{-t}) + kt^2e^{-t} \\ &= 2ke^{-t}.\end{aligned}$$

So $k = 1/2$ yields the solution

$$y_p(t) = \tfrac{1}{2}t^2e^{-t}.$$

From the characteristic polynomial, we know that the general solution of the unforced equation is

$$k_1e^{-t} + k_2te^{-t}.$$

Consequently, the general solution of the forced equation is

$$y(t) = k_1e^{-t} + k_2te^{-t} + \tfrac{1}{2}t^2e^{-t}.$$

20. If we guess a constant function of the form $y_p(t) = k$, then substituting $y_p(t)$ into the left-hand side of the differential equation yields

$$\begin{aligned}\frac{d^2(k)}{dt^2} + p\frac{d(k)}{dt} + qk &= 0 + 0 + qk \\ &= qk.\end{aligned}$$

Since the right-hand side of the differential equation is simply the constant c, $k = c/q$ yields a constant solution.

21. **(a)** The characteristic polynomial of the unforced equation is

$$s^2 - 5s + 4.$$

So the eigenvalues are $s = 1$ and $s = 4$, and the general solution of the unforced equation is

$$k_1 e^t + k_2 e^{4t}.$$

To find one solution of the forced equation, we guess the constant function $y_p(t) = k$. Substituting $y_p(t)$ into the left-hand side of the differential equation, we obtain

$$\frac{d^2 y_p}{dt^2} - 5\frac{dy_p}{dt} + 4y_p = 0 - 5 \cdot 0 + 4k = 4k.$$

Hence, $k = 5/4$ yields a solution of the forced equation. The general solution of the forced equation is

$$y(t) = k_1 e^t + k_2 e^{4t} + \tfrac{5}{4}.$$

(b) To find the solution satisfying the initial conditions $y(0) = y'(0) = 0$, we compute the derivative of the general solution

$$y'(t) = k_1 e^t + 4k_2 e^{4t}.$$

Using the initial conditions and evaluating $y(t)$ and $y'(t)$ at $t = 0$, we obtain the simultaneous equations

$$\begin{cases} k_1 + k_2 + \frac{5}{4} = 0 \\ k_1 + 4k_2 = 0. \end{cases}$$

Solving for k_1 and k_2 gives $k_1 = -5/3$ and $k_2 = 5/12$. The solution of the initial-value problem is

$$y(t) = \tfrac{5}{4} - \tfrac{5}{3}e^t + \tfrac{5}{12}e^{4t}.$$

25. **(a)** The characteristic polynomial of the unforced equation is

$$s^2 + 9.$$

So the eigenvalues are $s = \pm 3i$, and the general solution of the unforced equation is

$$k_1 \cos 3t + k_2 \sin 3t.$$

To find one solution of the forced equation, we guess $y_p(t) = ke^{-t}$. Substituting $y_p(t)$ into the left-hand side of the differential equation, we obtain

$$\begin{aligned} \frac{d^2 y_p}{dt^2} + 9y_p &= ke^{-t} + 9ke^{-t} \\ &= 10ke^{-t}. \end{aligned}$$

Hence, $k = 1/10$ yields a solution of the forced equation. The general solution of the forced equation is

$$y(t) = k_1 \cos 3t + k_2 \sin 3t + \tfrac{1}{10} e^{-t}.$$

(b) To find the solution satisfying the initial conditions $y(0) = y'(0) = 0$, we compute the derivative of the general solution

$$y'(t) = -3k_1 \sin 3t + 3k_2 \cos 3t - \tfrac{1}{10} e^{-t}.$$

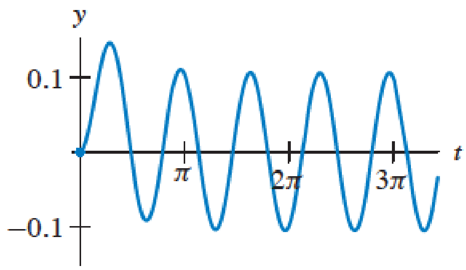
Using the initial conditions and evaluating $y(t)$ and $y'(t)$ at $t = 0$, we obtain the simultaneous equations

$$\begin{cases} k_1 + \frac{1}{10} = 0 \\ 3k_2 - \frac{1}{10} = 0. \end{cases}$$

Solving for k_1 and k_2 gives $k_1 = -1/10$ and $k_2 = 1/30$. The solution of the initial-value problem is

$$y(t) = -\tfrac{1}{10} \cos 3t + \tfrac{1}{30} \sin 3t + \tfrac{1}{10} e^{-t}.$$

(c) Since the function $e^{-t}/10 \to 0$ quickly, the solution quickly approaches a solution of the unforced oscillator.



31. **(a)** The general solution for the homogeneous equation is

$$k_1 \cos 2t + k_2 \sin 2t.$$

Suppose $y_p(t) = at^2 + bt + c$. Substituting $y_p(t)$ into the differential equation, we get

$$\frac{d^2y_p}{dt^2} + 4y_p = -3t^2 + 2t + 3$$

$$2a + 4(at^2 + bt + c) = -3t^2 + 2t + 3$$

$$4at^2 + 4bt + (2a + 4c) = -3t^2 + 2t + 3.$$

Therefore, $y_p(t)$ is a solution if and only if

$$\begin{cases} 4a = -3 \\ 4b = 2 \\ 2a + 4c = 3. \end{cases}$$

Therefore, $a = -3/4, b = 1/2$, and $c = 9/8$. The general solution is

$$y(t) = k_1 \cos 2t + k_2 \sin 2t - \tfrac{3}{4}t^2 + \tfrac{1}{2}t + \tfrac{9}{8}.$$

(b) To solve the initial-value problem, we use the initial conditions $y(0) = 2$ and $y'(0) = 0$ along with the general solution to form the simultaneous equations

$$\begin{cases} k_1 + \frac{9}{8} = 2 \\ 2k_2 + \frac{1}{2} = 0. \end{cases}$$

Therefore, $k_1 = 7/8$ and $k_2 = -1/4$. The solution is

$$y(t) = \tfrac{7}{8} \cos 2t - \tfrac{1}{4} \sin 2t - \tfrac{3}{4}t^2 + \tfrac{1}{2}t + \tfrac{9}{8}.$$

1. Recalling that the real part of e^{it} is $\cos t$, we see that the complex version of this equation is

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{it}.$$

To find a particular solution, we guess $y_c(t) = ae^{it}$. Then $dy_c/dt = iae^{it}$ and $d^2y_c/dt^2 = -ae^{it}$. Substituting these derivatives into the equation and collecting terms yields

$$(-a + 3ia + 2a)e^{-it} = e^{it},$$

which is satisfied if

$$(1 + 3i)a = 1.$$

Hence, we must have

$$a = \frac{1}{1+3i} = \frac{1}{10} - \frac{3}{10}i.$$

So

$$y_c(t) = \frac{1-3i}{10}e^{it} = \frac{1-3i}{10}(\cos t + i\sin t)$$

is a particular solution of the complex version of the equation. Taking the real part, we obtain the solution

$$y(t) = \tfrac{1}{10}\cos t + \tfrac{3}{10}\sin t.$$

To produce the general solution of the homogeneous equation, we note that the characteristic polynomial $s^2 + 3s + 2$ has roots $s = -2$ and $s = -1$. So the general solution is

$$y(t) = k_1e^{-2t} + k_2e^{-t} + \tfrac{1}{10}\cos t + \tfrac{3}{10}\sin t.$$

11. From Exercise 5, we know that the general solution of this equation is

$$y(t) = k_1e^{-4t} + k_2e^{-2t} + \tfrac{7}{85}\cos t + \tfrac{6}{85}\sin t.$$

To find the desired solution, we must solve for k_1 and k_2 using the initial conditions. We have

$$\begin{cases} k_1 + k_2 + \frac{7}{85} = 0 \\ -4k_1 - 2k_2 + \frac{6}{85} = 0. \end{cases}$$

We obtain $k_1 = 2/17$ and $k_2 = -1/5$. The desired solution is

$$y(t) = \tfrac{2}{17}e^{-4t} - \tfrac{1}{5}e^{-2t} + \tfrac{7}{85}\cos t + \tfrac{6}{85}\sin t.$$

15. **(a)** If we guess

$$y_p(t) = a\cos 3t + b\sin 3t,$$

then

$$y_p'(t) = -3a\sin 3t + 3b\cos 3t$$

and

$$y_p''(t) = -9a\cos 3t - 9b\sin 3t.$$

Substituting this guess and its derivatives into the differential equation gives

$$(-8a + 9b)\cos 3t + (-9a - 8b)\sin 3t = \cos 3t.$$

Thus $y_p(t)$ is a solution if a and b satisfy the simultaneous equations

$$\begin{cases} -8a + 9b = 1 \\ -9a - 8b = 0. \end{cases}$$

Solving these equations for a and b, we obtain $a = -8/145$ and $b = 9/145$, so

$$y_p(t) = -\tfrac{8}{145}\cos 3t + \tfrac{9}{145}\sin 3t$$

is a solution.

(b) If we guess

$$y_p(t) = A\cos(3t + \phi),$$

then

$$y_p'(t) = -3A\sin(3t + \phi)$$

and

$$y_p''(t) = -9A\cos(3t + \phi).$$

Substituting this guess and its derivatives into the differential equation yields

$$-8A\cos(3t + \phi) - 9A\sin(3t + \phi) = \cos 3t.$$

Using the trigonometric identities for the sine and cosine of the sum of two angles, we have

$$-8A\,(\cos 3t\cos\phi - \sin 3t\sin\phi) - 9A\,(\sin 3t\cos\phi + \cos 3t\sin\phi) = \cos 3t.$$

This equation can be rewritten as

$$(-8A\cos\phi - 9A\sin\phi)\cos 3t + (8A\sin\phi - 9A\cos\phi)\sin 3t = \cos 3t.$$

It holds if

$$\begin{cases} -8A\cos\phi - 9A\sin\phi = 1 \\ 9A\cos\phi - 8A\sin\phi = 0. \end{cases}$$

Multiplying the first equation by 9 and the second by 8 and adding yields

$$145A\sin\phi = -9.$$

Similarly, multiplying the first equation by -8 and the second by 9 and adding yields

$$145A\cos\phi = -8.$$

Taking the ratio gives

$$\frac{\sin\phi}{\cos\phi} = \tan\phi = \frac{9}{8}.$$

Also, squaring both $145A\sin\phi = -9$ and $145A\cos\phi = -8$ yields

$$145^2A^2\cos^2\phi + 145^2A^2\sin^2\phi = 145,$$

so $A^2 = 1/145$.

We can use either $A = 1/\sqrt{145}$ or $A = -1/\sqrt{145}$, but this choice of sign for A effects the value of ϕ. If we pick $A = -1/\sqrt{145}$, then $\sqrt{145}\,\sin\phi = 9$, $\sqrt{145}\,\cos\phi = 8$, and $\tan\phi = 9/8$. In this case, $\phi = \arctan(9/8)$. Hence, a particular solution of the original equation is

$$y_p(t) = \frac{1}{\sqrt{145}}\cos\left(3t + \arctan\frac{9}{8}\right).$$

17. Since p and q are both positive, the solution of the homogeneous equation (the unforced response) tends to zero. Hence, we can match solutions to equations by considering the period (or frequency) and the amplitude of the steady-state solution (forced response). We also need to consider the rate at which solutions tend to the steady-state solution.

(a) The steady-state solution has period $2\pi/3$, and since the period of the steady-state solution is the same as the period of the forcing function, these solutions correspond to equations (v) or (vi). Moreover, this observation applies to the solutions in part (d) as well. Therefore, we need to match equations (v) and (vi) with the solutions in parts (a) and (d).

Solutions approach the steady-state faster in (d) than in (a). To distinguish (v) from (vi), we consider their characteristic polynomials. The characteristic polynomial for (v) is

$$s^2 + 5s + 1,$$

which has eigenvalues $(-5 \pm \sqrt{21})/2$. The characteristic polynomial for (vi) is

$$s^2 + s + 1,$$

which has eigenvalues $(-1 \pm i\sqrt{3})/2$. The rate of approach to the steady-state for (v) is determined by the slow eigenvalue $(-5 + \sqrt{21})/2 \approx -0.21$. The rate of approach to the steady-state for (vi) is determined by the real part of the eigenvalue, -0.5. Therefore, the graphs in part (a) come from equation (v), and the graphs in part (d) come from equation (vi).

(b) The steady-state solution has period 2π, and since the period of the steady-state solution is the same as the period of the forcing function, these solutions correspond to equations (i) or (ii). Moreover, this observation applies to the solutions in part (c) as well. Therefore, we need to match equations (i) and (ii) with the solutions in parts (b) and (c).

The amplitude of the steady-state solution is larger in (b) than in (c). To distinguish (i) from (ii), we calculate the amplitudes of the steady-state solutions for (i) and (ii). If we complexify these equations, we get

$$\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = e^{it}.$$

Guessing a solution of the form $y_c(t) = ae^{it}$, we see that

$$a = \frac{1}{(q-1) + pi}.$$

The amplitude of the steady-state solution is $|a|$. For equation (i), $|a| = 1/\sqrt{29} \approx 0.19$, and for equation (ii), it is $1/\sqrt{5} \approx 0.44$. Therefore, the graphs in part (b) correspond to equation (ii), and the graphs in part (c) correspond to equation (i).

(c) See the answer to part (b).

(d) See the answer to part (a).

23. Note that the real part of

$$(k_1 - ik_2)e^{i\beta t} = (k_1 - ik_2)(\cos\beta t + i\sin\beta t)$$

is

$$y(t) = k_1\cos\beta t + k_2\sin\beta t.$$

Let $Ke^{i\phi}$ be the polar form of the complex number $k_1 + ik_2$. Then the polar form of $k_1 - ik_2$ is $Ke^{-i\phi}$. Using the Laws of Exponents and Euler's formula, we have

$$\begin{aligned}(k_1 - ik_2)e^{i\beta t} &= Ke^{-i\phi}e^{i\beta t}\\ &= Ke^{i(\beta t - \phi)}\\ &= K(\cos(\beta t - \phi) + i\sin(\beta t - \phi),\end{aligned}$$

and the real part is $K\cos(\beta t - \phi)$. Hence, we see that

$$y(t) = k_1\cos\beta t + k_2\sin\beta t$$

can be rewritten as

$$y(t) = K\cos(\beta t - \phi).$$

1. The complex version of this equation is

$$\frac{d^2y}{dt^2} + 9y = e^{it}.$$

Guessing $y_c(t) = ae^{it}$ as a particular solution and substituting this guess into the left-hand side of the differential equation yields

$$8ae^{it} = e^{it}.$$

Thus, $y_c(t)$ is a solution if $8a = 1$. The real part of

$$y_c(t) = \tfrac{1}{8}e^{it} = \tfrac{1}{8}(\cos t + i\sin t)$$

is $y(t) = \frac{1}{8}\cos t$. This $y(t)$ is a solution to the original differential equation. [Because there is no dy/dt-term (no damping), we could have guessed a solution of the form $y(t) = a\cos t$ instead of using the complex version of the equation.]

To find the general solution of the homogeneous equation, we note that the characteristic polynomial is $s^2 + 9$, which has roots $s = \pm 3i$. So the general solution of the original equation is

$$y(t) = k_1\cos 3t + k_2\sin 3t + \tfrac{1}{8}\cos t.$$

5. The complex version of the equation is

$$\frac{d^2y}{dt^2} + 9y = 2e^{3it}.$$

Guessing $y_c(t) = ae^{3it}$ as a particular solution and substituting this guess into the left-hand side of the differential equation, we see that a must satisfy

$$-9a + 9a = 2,$$

which is impossible. Hence, the forcing is in resonance with the associated homogeneous equation.

We must make a second guess of $y_c(t) = ate^{3it}$. This guess gives

$$y_c'(t) = a(1 + 3it)e^{3it}$$

and

$$y_c''(t) = a(6i - 9t)e^{3it}.$$

Substituting $y_c(t)$ and its second derivative into the differential equation, we obtain

$$a(6i - 9t)e^{3it} + 9ate^{3it} = 2e^{3it},$$

which simplifies to

$$6aie^{3it} = 2e^{3it}.$$

Thus, $y_c(t)$ is a solution if $a = 2/(6i) = -i/3$. Taking the real part of

$$y_c(t) = -\tfrac{1}{3}it(\cos 3t + i \sin 3t),$$

we obtain the solution

$$y(t) = \tfrac{1}{3}t \sin 3t$$

of the original equation.

To find the general solution of the homogeneous equation, we note that the characteristic polynomial is $s^2 + 9$, which has roots $s = \pm 3i$.

Hence, the general solution of the original equation is

$$y(t) = k_1 \cos 3t + k_2 \sin 3t + \tfrac{1}{3}t \sin 3t.$$

9. From Exercise 1, we know that the general solution is

$$y(t) = k_1 \cos 3t + k_2 \sin 3t + \tfrac{1}{8} \cos t.$$

So

$$y'(t) = -3k_1 \sin 3t + 3k_2 \cos 3t - \tfrac{1}{8} \sin t.$$

Using the initial conditions $y(0) = 0$ and $y'(0) = 0$, we obtain the simultaneous equations

$$\begin{cases} k_1 + \frac{1}{8} = 0 \\ 3k_2 = 0, \end{cases}$$

which imply that $k_1 = -1/8$ and $k_2 = 0$. The solution to the initial-value problem is

$$y(t) = -\tfrac{1}{8} \cos 3t + \tfrac{1}{8} \cos t.$$

13. From Exercise 5, we know that the general solution is

$$y(t) = k_1 \cos 3t + k_2 \sin 3t + \tfrac{1}{3} t \sin 3t.$$

So

$$y'(t) = -3k_1 \sin 3t + 3k_2 \cos 3t + \tfrac{1}{3} \sin 3t + t \cos 3t.$$

From the initial condition $y(0) = 2$, we see that $k_1 = 2$. Using the initial condition $y'(0) = -9$, we have $3k_2 = -9$. Hence, $k_2 = -3$. The solution to the initial-value problem is

$$y(t) = 2 \cos 3t - 3 \sin 3t + \tfrac{1}{3} t \sin 3t.$$

15. The characteristic polynomial of the unforced equation is $s^2 + 4$, which has roots $s = \pm 2i$. So the natural frequency is $2/(2\pi)$, and the forcing frequency is $9/(8\pi)$.

(a) The frequency of the beats is

$$\frac{\frac{9}{4} - 2}{4\pi} = \frac{1}{16\pi},$$

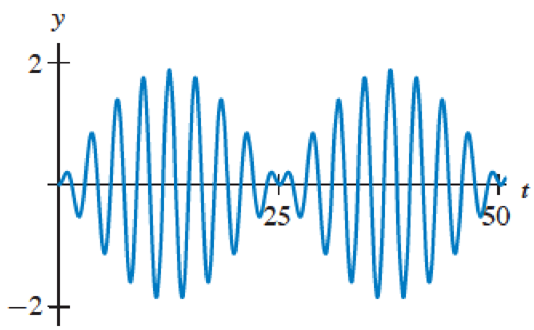
and therefore, the period of one beat is $16\pi \approx 50$.

(b) The frequency of the rapid oscillations is

$$\frac{\frac{9}{4} + 2}{4\pi} = \frac{17}{16\pi}.$$

Therefore, there are 17 rapid oscillations in each beat.

(c)



20. The crystal glass and the opera singer's voice have similar frequencies. The singer's voice becomes a driving force, and the glass is shattered due to resonance. If the recorded voice has the same effect on the glass, the recorded voice also has a frequency similar to the glass's frequency. Thus, the recorded sound must have a frequency that is very close to the frequency of the original sound.

21. **(a)** The graph shows either the solution of a resonant equation or one with beats whose period is very large. The period of the beats in equation (iii) is 4π, and the period of the beats in equation (iv) is $4\pi/(4-\sqrt{14}) \approx 48.6$. Hence this graph must correspond to a solution of the resonant equation—equation (v).

(b) The graph has beats with period 4π. Therefore, this graph corresponds to equation (iii).

(c) This solution has no beats and no change in amplitude. Therefore, it corresponds to either (i), (ii), or (vi). Note that the general solution of equation (i) is

$$k_1 \cos 4t + k_2 \sin 4t + \tfrac{5}{8},$$

and the general solution of equation (ii) is

$$k_1 \cos 4t + k_2 \sin 4t - \tfrac{5}{8}.$$

Equation (iv) has a steady-state solution whose oscillations are centered about $y = 0$. Since the oscillations shown are centered around a positive constant, this function is a solution of equation (i).

(d) The graph has beats with a period that is approximately 50. Therefore, this graph corresponds to equation (iv) (see part (a)).

22. The frequency of the stomping is almost the same as the natural frequency of the swaying motion of the stadium. Therefore, the stadium structure reacted violently due to the resonant effects of the stomping.

23. The equation of motion for the unforced mass-spring system is

$$\frac{d^2y}{dt^2} + 16y = 0,$$

so the natural period is $2\pi/4 = \pi/2 \approx 1.57$.

Tapping with the hammer as shown increases the velocity if the mass is moving to the right at the time of the tap and decreases the velocity if the mass is moving to the left at the time of the tap. Faster motion results in higher amplitude oscillations. Since none of the tapping periods is exactly $\pi/2$, the taps sometimes increase the amplitude and sometimes decrease the amplitude of the oscillations (that is, resonance does not occur).

The period $T = 3/2$ is closest to the natural period and hence for taps with this period we expect the largest amplitude oscillations.

24. To produce the most dramatic effect, the forcing frequency due to the speed bumps must agree with the natural frequency of the suspension system of the average car. Therefore, the speed bumps should be spaced so that the amount of time between bumps is exactly the same as the natural period of the oscillator. Since the natural period of the oscillator is 2 seconds, we compute the distance that the car travels in 2 seconds. At 10 miles per hour, the car travels $1/180$ miles in 2 seconds, and $1/180$ miles is 29 feet, 4 inches.