Mathematica Problems

8 Problems to Illustrate the utility of *Mathematica*
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Problem 1: Solve a Differential Equation (Adapted from Chapter 1.5, #12)

```
DSolve[y'[t] == -y[t]^2, y, t]
```

$$\left\{\left\{y \to \text{Function}\left[\{t\}, \frac{1}{t - C[1]}\right]\right\}\right\}$$

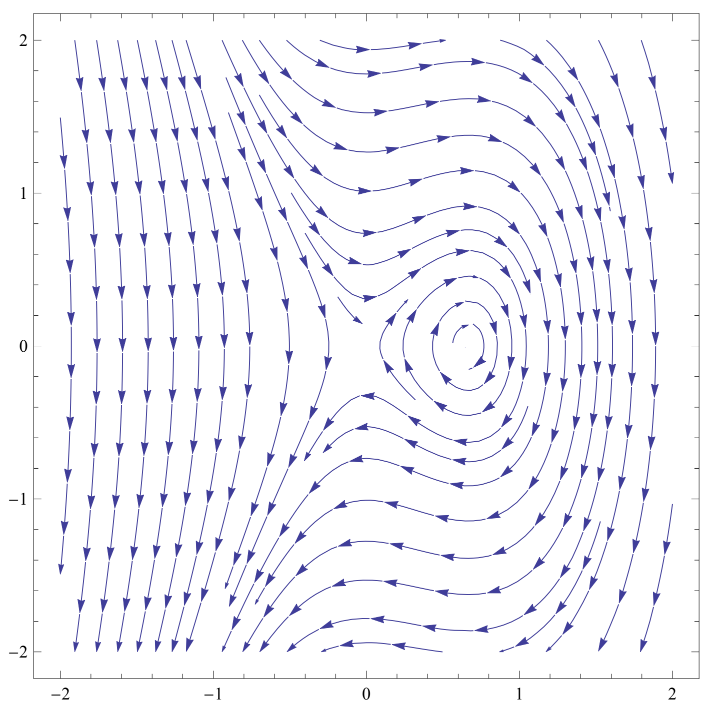
Problem 2: Solve an Inital Value Problem (Chapter 1 Review, #30)

```
DSolve[{x'[t] == -2 t x[t], x[0] == E}, x, t]
```

$$\left\{\left\{x \to \text{Function}\left[\{t\}, e\, e^{-t^2}\right]\right\}\right\}$$

Problem 3: Draw a Slope Field for a Differential Equation (Chapter 2 Review, #23)

```
StreamPlot[{y, 2 x - 3 x^2}, {x, -2, 2}, {y, -2, 2}]
```



Problem 4: Solve a system of Differential Equations (Adapted from Chapter 2.4 #5)

```
DSolve[{x'[t] == 2 x[t] + y[t], y'[t] == -y[t]}, {x, y}, t]
```

$$\left\{\left\{x \to \text{Function}\left[\{t\}, e^{2t} C[1] + \frac{1}{3} e^{-t} \left(-1 + e^{3t}\right) C[2]\right], y \to \text{Function}\left[\{t\}, e^{-t} C[2]\right]\right\}\right\}$$

Problem 5: Solve a system of Differential Equations for a given intial value (Adapted from Chapter 2.5 #3)

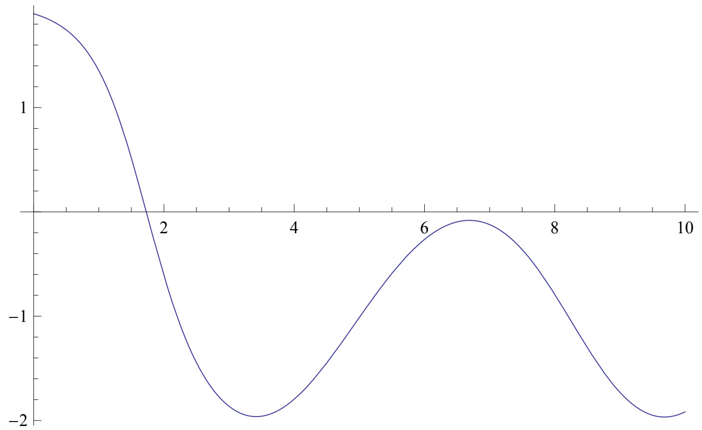
```
DSolve[{x'[t] == y[t], y'[t] == -2 x[t] - 3 y[t], x[0] == 1, y[0] == 1}, {x, y}, t]
```

$\{\{x \to \text{Function}[\{t\}, e^{-2t}(-2+3e^{t})], y \to \text{Function}[\{t\}, -e^{-2t}(-4+3e^{t})]\}\}$

Problem 6: Numerically solve a Differential Equation for a given intial value (Chapter 1 Review #43)

```
h = NDSolve[{y'[t] == (y[t] - 2) (y[t] + 1 - Cos[t]), y[0] == 1.9}, y, {t, 0, 10}]
Plot[y[t] /. h, {t, 0, 10}]
```

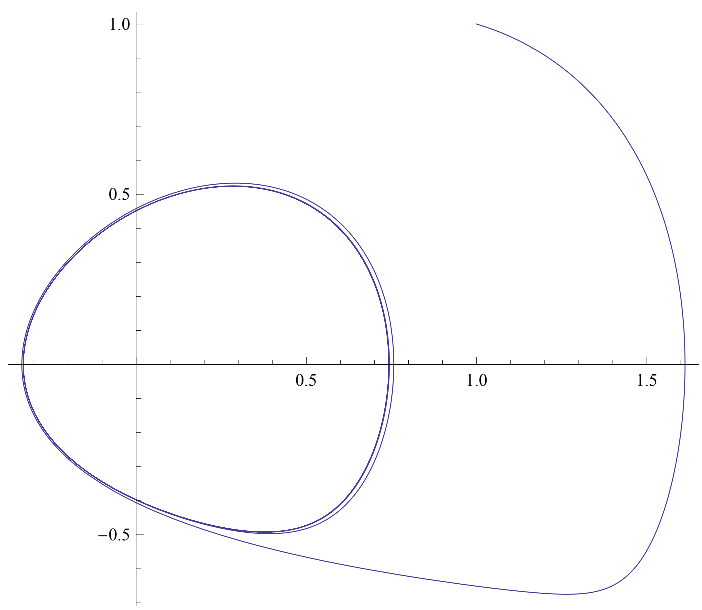
```
{{y → InterpolatingFunction[{{0., 10.}}, <>]}}
```



Problem 7: Numerically solve a system of Differential Equations (Chapter 2.5 #5)

```
g = NDSolve[{x'[t] == y[t] + y[t]^2, y'[t] == -x[t] + y[t] / 5 - (x[t] * y[t]) + (6 / 5 * y[t]^2),
   x[0] == 1, y[0] == 1}, {x, y}, {t, 0, 40}]
ParametricPlot[Evaluate[{x[t], y[t]} /. g], {t, 0, 40}]
```

```
{{x → InterpolatingFunction[{{0., 40.}}, <>], y → InterpolatingFunction[{{0., 40.}}, <>]}}
```



Problem 8: Solve a Differential Equation you cannot solve analytically (Lacked example I wanted, so made up)

```
DSolve[y'[t] == y[t]^2 * t + 1, y, t]
```

$$\left\{\left\{y \to \text{Function}\left[\{t\}, -\frac{(-1)^{2/3}\, t\, \text{AiryBi}\left[(-1)^{1/3} t\right] + (-1)^{2/3}\, t\, \text{AiryAi}\left[(-1)^{1/3} t\right] C[1]}{t \left(\text{AiryBiPrime}\left[(-1)^{1/3} t\right] + \text{AiryAiPrime}\left[(-1)^{1/3} t\right] C[1]\right)}\right]\right\}\right\}$$

Wikipedia Informs me that this function makes heavy use of the Airy function, which is defined as the solution to y''[t] = y*t. Since we're in the realm of functions which are created to solve problems, rather than those which are found to solve problems, I imagine this branch of differential equations is beyond the scope of our class.