MAT5-236 Differential Equations
Final Exam Take-home part February 2, 2018

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name

This part of the exam is take-home—you are free to use any written materials (but do not plagiarize) and **you must use software to solve these problem**s. Do all the work yourself --no consultation with any other individuals; questions should come to me. Use this page as a cover sheet for your answers. Due 5 p.m. Tuesday February 6. Please hand in your solutions to me in class or at my office (under the door is OK).

1 (8 pts.) a. Use Laplace transforms to solve $y' + y = 2u_1(t)$, $y(0) = 1$, where u is the Heaviside function.

b. Use the software provided with our text to produce a phase portrait for this equation. (Include the printed portrait.)

c. Use your answer to a. and the phase portrait to describe the behavior of the solution as t goes to infinity.

2. (4 pts) Solve $y' = \begin{matrix} 4 & 1 & 4 \\ 1 & 7 & 1 \\ 4 & 1 & 4 \end{matrix} y$

3. (6 pts.) Using software, sketch the phase portrait of the following system, restricted to the first quadrant. On that graph, sketch the nullclines. Write a couple sentences describing possible long-term behavior of solutions.
$x' = x(-4x-y+160)$
$y' = y(-x^2-y^2+2500)$

4. (7 pts.) Suppose you throw a ball into the air. Does it take longer going up, or coming back down to where it started or does it take the same time? If a ball has a smooth surface and low velocity, this physical system can be modeled in one way. If it is a whiffle ball, then it is better to consider a friction term (this is called viscous damping—proportional to velocity and acting in the opposite direction). Answer the question for both situations basing your analysis on graphs of the solution curve. (Use Excel or Mathematica or our text software—your choice).