# Modeling Fractions

## CSC144 Software Architecture

### 19 March 2018

# 1 Arithmetic with fractions

## 1.1 Greatest common divisor

A fraction can be reduced to its simplest form by dividing numerator and denominator by the greatest common divisor of both.

The greatest common divisor of two non-negative integers can be found with a recursive algorithm:

$$gcd(a, b) = gcd(b, a \bmod b)$$
$$gcd(a, 0) = a$$

### 1.1.1 Example

$$
\begin{aligned}
gcd(144, 60) &= gcd(60, 144 \bmod 60) \\
&= gcd(60, 24) \\
&= gcd(24, 60 \bmod 24) \\
&= gcd(24, 12) \\
&= gcd(12, 24 \bmod 12) \\
&= gcd(12, 0) \\
&= 12
\end{aligned}
$$

## 1.2 Addition

$$a = \frac{num_a}{den_a}$$

$$b = \frac{num_b}{den_b}$$

$$a + b = \frac{num_a \cdot den_b + num_b \cdot den_a}{den_a \cdot den_b}$$

### 1.2.1 Example

$$\frac{2}{5} + \frac{3}{8} = \frac{2 \cdot 8 + 3 \cdot 5}{5 \cdot 8}$$
$$= \frac{31}{40}$$

## 1.3 Subtraction

$$a = \frac{num_a}{den_a}$$

$$b = \frac{num_b}{den_b}$$

$$a - b = \frac{num_a \cdot den_b - num_b \cdot den_a}{den_a \cdot den_b}$$

### 1.3.1 Example

$$\frac{2}{5} + \frac{3}{8} = \frac{2 \cdot 8 - 3 \cdot 5}{5 \cdot 8}$$

$$= \frac{1}{40}$$

## 1.4 Multiplication

$$a = \frac{num_a}{den_a}$$

$$b = \frac{num_b}{den_b}$$

$$a \times b = \frac{num_a \cdot num_b}{den_a \cdot den_b}$$

### 1.4.1 Example

$$\frac{2}{5} \times \frac{3}{8} = \frac{2 \cdot 3}{5 \cdot 8}$$

$$= \frac{6}{40}$$

$$= \frac{3}{20}$$

## 1.5 Division

$$a = \frac{num_a}{den_a}$$

$$b = \frac{num_b}{den_b}$$

$$a \div b = \frac{num_a \cdot den_b}{den_a \cdot num_b}$$

### 1.5.1 Example

$$\frac{3}{8} \div \frac{2}{5} = \frac{3 \cdot 5}{8 \cdot 2}$$

$$= \frac{15}{16}$$

# 2   A Java class that models a fraction

```java
package fraction;

public class Fraction {
    private final int numerator;
    private final int denominator;

    public Fraction( int numerator, int denominator ) {
        int divisor = gcd( numerator, denominator );
        this.numerator = numerator/divisor;
        this.denominator = denominator/divisor;
    } // Fraction( int, int )

    public int getNumerator() {
        return this.numerator;
    } // getNumerator()

    public int getDenominator() {
        return this.denominator;
    } // getDenominator()

    public Fraction add( Fraction otherFraction ) {
        int n0 = this.getNumerator();
        int d0 = this.getDenominator();

        int n1 = otherFraction.getNumerator();
        int d1 = otherFraction.getDenominator();

        return new Fraction( n0 * d1 + n1 * d0, d0 * d1 );
    } // add( Fraction )

    public Fraction subtract( Fraction otherFraction ) {
        int n0 = this.getNumerator();
        int d0 = this.getDenominator();
```

```java
        int n1 = otherFraction.getNumerator();
        int d1 = otherFraction.getDenominator();

        return new Fraction( n0 * d1 - n1 * d0, d0 * d1 );
} // subtract( Fraction )

public Fraction multiply( Fraction otherFraction ) {
        int n0 = this.getNumerator();
        int d0 = this.getDenominator();

        int n1 = otherFraction.getNumerator();
        int d1 = otherFraction.getDenominator();

        return new Fraction( n0 * n1, d0 * d1 );
} // multiply( Fraction )

public Fraction divide( Fraction otherFraction ) {
        int n0 = this.getNumerator();
        int d0 = this.getDenominator();

        int n1 = otherFraction.getNumerator();
        int d1 = otherFraction.getDenominator();

        return new Fraction( n0 * d1, d0 * n1 );
} // divide( Fraction )

@Override
public String toString() {
        return this.getNumerator() + "/" + this.getDenominator();
} // toString()

private int gcd( int a, int b ) {
        if( b == 0 ) {
            return a;
        } // if
        else {
            return gcd( b, a % b );
        } // else
} // gcd( int, int )

public static void main( String [] args ) {
        Fraction a = new Fraction(2,5);
        Fraction b = new Fraction(3,8);

        System.out.println( "a = " + a );
```

```java
        System.out.println( "b = " + b );

        System.out.println( "a + b = " + a.add(b) );
        System.out.println( "a - b = " + a.subtract(b) );
        System.out.println( "a * b = " + a.multiply(b) );
        System.out.println( "b / a = " + b.divide(a) );
    } // main( String [] )

} // Fraction
```